Gauge theories without Higgs boson

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Abstract.

A new formulation of the Electroweak Model with 3-dimensional spherical geometry in the target space is suggested. The free Lagrangian in the spherical field space along with the standard gauge field Lagrangian form the full Higgsless Lagrangian of the model, whose second order terms reproduce the same fields with the same masses as the Standard Electroweak Model. The vector bosons are automatically generated, so there is no need in special mechanism of spontaneous symmetry breaking.

1 Introduction

The Standard Electroweak Model (SEWM) based on gauge group $SU(2) \times U(1)$ gives a good description of electroweak processes. One of the unsolved problems is the origin of electroweak symmetry breaking. In the standard formulation the scalar field (Higgs boson) performs this task via Higgs mechanism, which generates a mass terms for vector bosons. However, it is not yet experimentally verified whether electroweak symmetry is broken by such a Higgs mechanism, or by something else.

The emergence of large number Higgsless models [1]–[7] was stimulated by difficulties with Higgs boson. These models are mainly based on extra dimensions of different types or larger gauge groups. The construction given in [8] is based on an observation: the underlying group of SEWM can be represented as a semidirect product of $U(1)$ and $SU(2)$.

In the present paper a new formulation of the Higgsless Electroweak Model is suggested. Firstly we observe that the quadratic form $\phi^\dagger \phi = \phi_1^* \phi_1 + \phi_2^* \phi_2 = R^2$ of the complex matter field $\phi \in C_2$ is invariant with respect to gauge group transformations $SU(2) \times U(1)$ and we can restrict fields on the quadratic form without the loss of gauge invariance of the model. This quadratic form define three dimensional sphere in four dimensional Euclidean space of the real components of $\phi$, where the noneuclidean spherical geometry is realized.

Secondly we introduce the free matter field Lagrangian in this spherical field space, which along with the standard gauge field Lagrangian form the full Higgsless Lagrangian of the model. Its second order terms reproduce the same fields as the SEWM but without the remaining real dynamical Higgs field. The vector bosons masses are automatically generated and are given by the same formulas as in the SEWM, so there is no need in special mechanism of spontaneous symmetry breaking.
The fermion Lagrangian of the SEWM are modified by replacing of the fields $\phi$ with the restricted on the quadratic form fields in such a way that its second order terms provide the electron mass and neutrino remain massless. The preliminary versions are in [9],[10].

## 2 Standard Electroweak Model

The bosonic sector of SEWM is $SU(2) \times U(1)$ gauge theory in the space $\Phi_2(\mathbb{C})$ of fundamental representation of $SU(2)$. The bosonic Lagrangian is given by the sum $L_B = L_A + L_\phi$, where

$$L_A = \frac{1}{8g^2} \text{Tr}(F_{\mu\nu})^2 - \frac{1}{4}(B_{\mu\nu})^2 = -\frac{1}{4}[(F_{\mu\nu}^1)^2 + (F_{\mu\nu}^2)^2 + (F_{\mu\nu}^3)^2] - \frac{1}{4}(B_{\mu\nu})^2 \tag{1}$$

is the gauge field Lagrangian for $SU(2) \times U(1)$ group and

$$L_\phi = \frac{1}{2}(D_\mu \phi)^\dagger D_\mu \phi - \frac{\lambda}{4} (\phi^\dagger \phi - v^2)^2 \tag{2}$$

is the matter field Lagrangian with the by hand introduced potential of the special form (sombrero), where $\lambda, v$ are constants (summation on the repeating Greek indexes is always understood). Here $D_\mu$ are the covariant derivatives

$$D_\mu \phi = \partial_\mu \phi - ig \left( \sum_{k=1}^3 T_k A^k_\mu \right) \phi - ig' Y B_\mu \phi, \tag{3}$$

where $T_k = \frac{i}{2} \sigma_k, k = 1, 2, 3$, with $\sigma_k$ being Pauli matrices, are linear representation of generators in the complex space $\Phi_2(\mathbb{C})$ of the fundamental representation of $SU(2)$ and $Y = \frac{1}{2} \mathbf{1}$ is generator of $U(1)$ in the same space. The stress tensors are $F_{\mu\nu}(x) = \mathcal{F}_{\mu\nu}(x) + [A_\mu(x), A_\nu(x)], B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.

The Lagrangian $L_B$ describe massless fields. To generate mass terms for the vector bosons without breaking the gauge invariance one uses the Higgs mechanism. One of $L_B$ ground states $\phi^{vac} = \begin{pmatrix} 0 \\ v \end{pmatrix}, A^k_\mu = B_\mu = 0$ is taken as a vacuum state of the model and small field excitations $\phi_1(x), \phi_2(x) = v + \chi(x), A^a_\mu(x), B_\mu(x)$ with respect to the vacuum are regarded. The new fields

$$W^\pm_\mu = \frac{1}{\sqrt{2}} (A^1_\mu \mp i A^2_\mu), \quad Z_\mu = \frac{g A^3_\mu - g' B_\mu}{\sqrt{g^2 + g'^2}}, \quad A_\mu = \frac{g' A^3_\mu + g B_\mu}{\sqrt{g^2 + g'^2}}$$

are introduced, where $W^\pm_\mu$ are complex ($W^-_\mu = W^+_\mu$ and $Z_\mu, A_\mu$ are real.

For small fields the Lagrangian $L_B$ can be rewritten in the form $L_B = L_0 + L_{int}$, where $L_0 = L_B^{(2)}$ is the usual second order Lagrangian for free vector and scalar bosons, and higher order terms $L_{int}$ are regarded as field interactions. The second order terms of $L_B$ are as follows

$$L_B^{(2)} = -\frac{1}{2} W^+_\mu W^-_\mu + m^2 W^+_\mu W^-_\mu - \frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} m^2 Z_{\mu \nu} Z_{\mu \nu} + \frac{1}{2} (\partial_\mu \chi)^2 - \frac{1}{2} m^2 \chi^2, \tag{4}$$
where \( Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \), \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), \( W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm \) and describe
massive vector fields \( W_\mu^\pm \) with identical mass \( m_W = \frac{1}{2} g v \) (W-bosons), massless vector
field \( A_\mu \), \( m_A = 0 \) (photon), massive vector field \( Z_\mu \), with the mass \( m_Z = \frac{\sqrt{2}}{2} \sqrt{g^2 + g'^2} \)
(Z-boson) and massive scalar field \( \chi \), \( m_\chi = \sqrt{2\lambda v} \) (Higgs boson).

W- and Z-bosons have been observed and have the masses \( m_W = 80\text{GeV} \), \( m_Z = 91\text{GeV} \). Higgs boson has not been experimentally verified up to now.

### 3 Higgsless Electroweak Model with 3D spherical matter space

The complex 2D space \( \Phi_2 \) can be regarded as 4D real Euclidean space \( \mathbb{R}_4 \). Let us introduce
the real fields \( r, \bar{\psi} = (\psi_1, \psi_2, \psi_3) \) by \( \phi_1 = r(\psi_2 + i\bar{\psi}_1) \), \( \phi_2 = r(1 + i\bar{\psi}_3) \). It is easy to see
that the quadratic form
\[
\phi^\dagger \phi = \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 = R^2
\]
is invariant with respect to gauge group transformations \( SU(2) \times U(1) \) and we can restrict
fields on this quadratic form without the loss of gauge invariance of the model. Similar
restriction was appeared in the unified conformal model for fundamental interactions [11]
in the case when the length scale is defined in such a way that elementary particle masses
are the same for all times and in all places.

For the real fields the form (5) is written as \( r^2(1 + \bar{\psi}^2) = R^2 \), where \( \bar{\psi}^2 = \psi_1^2 + \psi_2^2 + \psi_3^2 \),
therefore \( r = R(1 + \bar{\psi}^2)^{-\frac{1}{2}} \). Hence there are three independent real fields \( \bar{\psi} \). These fields
belong to the space \( \Psi_3 \) with noneuclidean spherical geometry which is realized on the 3D
sphere in 4D Euclidean space \( \mathbb{R}_4 \). The fields \( \bar{\psi} \) are intrinsic Beltrami coordinates on \( \Psi_3 \).

As the next step, let us define the free gauge invariant matter field Lagrangian \( L_\psi \)
with the help of the metric tensor [9] of the spherical space \( \Psi_3 \) in the form
\[
L_\psi = \frac{R^2}{2} \sum_{k,l=1}^3 g_{kl}D_\mu \psi_k D_\mu \psi_l = \frac{R^2 [(1 + \bar{\psi}^2)(D_\mu \bar{\psi})^2 - (\bar{\psi}, D_\mu \bar{\psi})^2]}{2(1 + \psi^2)^2}.
\]

Let us note that the Lagrangian on 3D sphere in 4D Euclidean space was considered in
[12], where the invariant perturbative theory was developed which does not depend on
some particular parametrization. The covariant derivatives (3) are obtained using the
nonlinear representations of generators for the algebras \( su(2), u(1) \) in the space \( \Psi_3 \) [9]
and are as follows:
\[
D_\mu \psi_1 = \partial_\mu \psi_1 + \frac{g}{2} [- (1 + \psi_2^2) A^1_\mu - (\psi_3 + \psi_1 \psi_2) A^2_\mu - (\psi_2 - \psi_1 \psi_3) A^3_\mu] - \frac{g'}{2} (\psi_2 + \psi_1 \psi_3) B_\mu,
\]
\[
D_\mu \psi_2 = \partial_\mu \psi_2 + \frac{g}{2} [(\psi_3 - \psi_1 \psi_2) A^1_\mu - (1 + \psi_2^2) A^2_\mu + (\psi_1 + \psi_2 \psi_3) A^3_\mu] + \frac{g'}{2} (\psi_1 - \psi_2 \psi_3) B_\mu,
\]
\[
D_\mu \psi_3 = \partial_\mu \psi_3 + \frac{g}{2} [- (\psi_2 + \psi_1 \psi_3) A^1_\mu + (\psi_1 - \psi_2 \psi_3) A^2_\mu + (1 + \psi_3^2) A^3_\mu] - \frac{g'}{2} (1 + \psi_2^2) B_\mu.
\]
The gauge fields Lagrangian (1) does not depend on \( \phi \) and therefore remains unchanged.

For small fields, the second order part of the Lagrangian (6) is written as
\[
L_\psi^{(2)} = \frac{R^2}{2} [ (D_\mu \bar{\psi})^{(1)} ]^2 = \frac{R^2}{2} \sum_{k=1}^3 [ (D_\mu \psi_k)^{(1)} ]^2,
\]
where linear terms in covariant derivate (7) have the form

\[(D_\mu \psi_1)^{(1)} = -\frac{g}{2} \left( A^1_\mu - \frac{2}{g} \partial_\mu \psi_1 \right) = -\frac{g}{2} \hat{A}^1_\mu, \quad (D_\mu \psi_2)^{(1)} = -\frac{g}{2} \left( A^2_\mu - \frac{2}{g} \partial_\mu \psi_2 \right) = -\frac{g}{2} \hat{A}^2_\mu, \quad (D_\mu \psi_3)^{(1)} = \partial_\mu \psi_3 + \frac{g}{2} A^3_\mu - \frac{g}{2} B_\mu = \frac{1}{2} \sqrt{g^2 + g'^2} Z_\mu. \]

For the new fields

\[W^\pm_\mu = \frac{1}{\sqrt{2}} \left( \hat{A}^1_\mu \mp i \hat{A}^2_\mu \right), \quad Z_\mu = \frac{g A^3_\mu - g' B_\mu + 2 \partial_\mu \psi_3}{\sqrt{g^2 + g'^2}}, \quad A_\mu = \frac{g' A^3_\mu + g B_\mu}{\sqrt{g^2 + g'^2}}\]

the quadratic part of the full Lagrangian (1) and (8) is rewritten as follows

\[L_0 = L_A^{(2)} + L_\psi^{(2)} = -\frac{1}{2} W^\pm_\mu W_\mu^\pm + m^2_W W^+ \bar{W}^- - \frac{1}{4} (F_{\mu\nu})^2 - \frac{1}{4} (Z_{\mu\nu})^2 + \frac{m^2_{\tilde{Z}}}{2} (Z_{\mu})^2,\]

where \(m_W = \frac{R_W}{2}, \quad m_{\tilde{Z}} = \frac{\sqrt{2}}{2} \sqrt{g^2 + g'^2}, \quad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad W^\pm_\mu = \partial_\mu W^\pm_\mu - \partial_\nu W^\pm_{\mu\nu} \) and describes all the experimentally verified parts of SEWM but does not include the dynamical scalar Higgs field. For \(R = v\) these masses are identical to those of the SEWM (4).

The fermion Lagrangian of SEWM is taken in the form [13]

\[L_F = L^1_i i\bar{\sigma}_\mu D_\mu L_i + e^\dagger_i i\sigma_\mu D_\mu e_r - h_c \left[ e^\dagger_i (\phi^\dagger L_i) + (L^1_i \phi) e_r \right], \quad (9)\]

where \(L_i = \begin{pmatrix} \nu_{ei} \\ e_i \end{pmatrix}\) is the \(SU(2)\)-doublet, \(e_i\) the \(SU(2)\)-singlet, \(h_c\) is constant and \(e_r, e_l, \nu_e\) are two component Lorentzian spinors. The matter fields \(\phi\) appear in Lagrangian (9) only in mass terms. For the independent real fields \(\bar{\psi}\) these mass terms are rewritten in the form

\[h_c \left[ e^\dagger_i (\phi^\dagger L_i) + (L^1_i \phi) e_r \right] = \frac{h_c R}{\sqrt{1 + \psi^2}} \left[ e^\dagger_i e_i^{-} + e_i^{-} e_r + \right. \left. + i\psi_3 \left( e_i^{-} e_r - e^\dagger_i e_i^{-} \right) + i\psi_1 \left( \nu^\dagger_{ei} e_r - e^\dagger_i \nu_{ei} \right) + i\psi_2 \left( \nu^\dagger_{ei} e_r + e^\dagger_i \nu_{ei} \right) \right].\]

Its second order terms \(h_c R \left( e^\dagger_i e_i^{-} + e_i^{-} e_r \right)\) provide the electron mass \(m_e = h_c R\), and neutrino remain massless.

4 Conclusion

The suggested formulation of the Electroweak Model with the gauge group \(SU(2) \times U(1)\) based on the 3-dimensional spherical geometry in the target space describes all experimentally observed fields, and does not include the (up to now unobserved) scalar Higgs field. The free Lagrangian in the spherical matter field space is used instead of Lagrangian (2) with the by hand introduced potential of the special form (sombrero). The gauge field Lagrangian is the standard one. There is no need in Higgs mechanism since the vector field masses are automatically generated. The renormalizability of the model is the open question at present and will be a subject of further investigations.

4
The motion group of the spherical space $\Psi_3$ is isomorphic to $SO(4)$. It is known [14], that the bosonic sector of SEWM has the custodial $SU(2)$-symmetry, and potential in (2) is globally invariant with respect of $SO(4)$ which is locally isomorphic to the group $SU(2)_L \times SU(2)_R$, where $SU(2)_L$ is the gauge group of SEWM. The appearance of the extra $SU(2)_R$-symmetry is interpreted as transformation of $SU(2)$-doublet by pseudoreal representation, that is the complex conjugate doublet is equivalent to the initial one. The introduction of the real fields $\bar{\psi}$ is connected in a certain sense with $SO(4)$-symmetry.

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References


