Asymmetrical tunneling in heavy fermion metals as a possible probe for their non-Fermi liquid peculiarities

V.R. Shaginyan\textsuperscript{a,b,*}, K.G. Popov\textsuperscript{a,b}, V.A. Stephanovich\textsuperscript{c}, E.V. Kirichenko\textsuperscript{c}

\textsuperscript{a} Petersburg Nuclear Physics Institute, RAS, Gatchina 188300, Russia
\textsuperscript{b} Komi Science Center, Ural Division, RAS, Syktyvkar 167982, Russia
\textsuperscript{c} Institute of Mathematics and Informatics, Opole University, 45-052 Opole, Poland

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Abstract

Tunneling conductivity and point contact spectroscopy between heavy fermion metal and a simple metallic point contact may serve as a convenient probing tool for non-Fermi liquid behavior. Landau Fermi liquid theory predicts that the differential conductivity is a symmetric function of voltage bias. This symmetry, in fact, holds if so called particle–hole symmetry is preserved. Here, we show that the situation can be different when one of the two metals is a heavy fermion one whose electronic system is a heavy fermion liquid. When the heavy fermion liquid undergoes fermion condensation quantum phase transition, the particle–hole symmetry in the excitation spectra is violated making both the differential tunneling conductivity and dynamic conductance asymmetric as a function of applied voltage. This asymmetry can be observed when the heavy fermion metal is either normal or superconducting. We discuss also the possible experiments to study the above asymmetry and note that asymmetric conductivity has been recently observed in measurements on CeCoIn\textsubscript{5}.

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1. Introduction

The experiments on heavy fermion (HF) metals and high-$T_c$ superconductors (HTSC) explore mainly their thermodynamic properties. It is highly desirable to probe the other properties of heavy electron liquid like quasiparticle occupation numbers, which are not directly linked to the density of states or to the behavior of the effective mass $M^\ast$. Both scanning tunneling microscopy (STM) and point contact spectroscopy (PCS) being sensitive to both the density of states and quasiparticle occupation numbers are ideal techniques to study the effects of particle–hole symmetry violation, making the differential tunneling conductivity and dynamic conductance to be asymmetric function of applied voltage. This asymmetry can be observed when HF metals and HTSC are either normal or superconducting. We note that in the case of Landau Fermi liquid (LFL) the particle–hole symmetry conserves and both the differential tunneling conductivity and dynamic conductance are symmetric functions of voltage bias. Thus, STM and PCS provide a new direction in the experimental studies of the NFL behavior of HTSC and HF metals.

In this paper, we show that a particle–hole symmetry is violated when the heavy electron liquid undergoes fermion condensation quantum phase transition (FCQPT). As a result, both the differential tunneling conductivity and the dynamic conductance become asymmetric as a function of voltage $V$. The application of magnetic field destroys the non-Fermi liquid (NFL) behavior of the heavy electron liquid and restores the above symmetry.

2. Heavy electron liquid with fermion condensate

Now we briefly describe the heavy electron liquid with the fermion condensate (FC) \cite{3,4}. When the density $x$ of Fermi
liquid approaches some threshold value \(x_{FC}\) the effective mass diverges as \(M^*(x) = M/(1 - N_0F^1(p_F, p_F)/3)\). Here, \(N_0\) is the density of states of free electron gas, \(M\) is the electron mass, \(p_F\) is the Fermi momentum and \(F^1(p_F, p_F)\) is the \(P\)-wave component of the Landau interaction. Since in the LFL theory the number density \(x = p_F^2/3\pi^2\), the Landau amplitude can be written as \(F^1(p_F, p_F) = F^1(x)\). The effective mass diverges \([5]\) at the critical point \(x_{FC}\) since the denominator \((1 - N_0F^1(x)/3)\) tends to zero, and we obtain that \(M^*(x)\) behaves as \(M^*(x)/M \propto 1/r\). Here, \(r = (x - x_{FC})\) is the “distance” from FCQPT taking place at \(x_{FC}\). Beyond \(x_{FC}\) the distance \(r\) becomes negative making the effective mass negative. To escape the possibility of being in unstable and meaningless states with the negative effective mass, the system is to undergo FCQPT at \(x_{FC}\) and the stepwise quasiparticle distribution function \(n_p(p)\) does not deliver the minimum to the Landau functional \(E[n(p)]\). As a result, at \(T = 0\) and \(x < x_{FC}\) the quasiparticle distribution is determined by the standard equation for the minimum of a functional \([6]\)

\[
\frac{\delta E[n(p)]}{\delta n(p, T = 0)} = \epsilon(p) = \mu; \quad p_i \leq p \leq p_F. \tag{1}
\]

Eq. (1) determines the quasiparticle distribution function \(n_0(p)\) minimizing the ground state energy \(E \equiv E[n(p)]\). Being determined by Eq. (1), the function \(n_0(p)\) does not coincide with the step function \(\theta(p - p_F)\) in the region \((p_F - p_i)\), so that \(0 < n_0(p) < 1\), while outside the region it coincides with \(\theta(p - p_F)\). It follows from Eq. (1) that the single-particle spectrum \(\epsilon(p)\) is completely flat in this region. Such a state was called the state with FC since the quasiparticles in the range \(p_F - p_i\) of momentum space are confined to a single plane perpendicular to the Fermi momentum \(p_F\). The possible solution \(n_0(p)\) of Eq. (1) and the corresponding single-particle spectrum \(\epsilon(p)\) are shown in Fig. 1. At \(T = 0\), the relevant order parameter is the superconducting-like, \(\epsilon(p) = \sqrt{n_0(p)(1 - n_0(p))}\), with the entropy \(S = 0\). At \(0 < T\), the ordered state is destroyed and the entropy is given by the familiar expression

\[
S(T) = -2 \int [n(p, T)\ln n(p, T) + (1 - n(p, T))\ln(1 - n(p, T))] \frac{dp}{2\pi^2}. \tag{2}
\]

Inserting the solutions \(n_0(p)\) into Eq. (2) we obtain that the entropy contains a temperature independent part \(S_0/\pi \sim (p_F - p_i)/p_F\). At \(T < T_f\), it can be approximated as \(S(T) \approx S_0 + a\sqrt{T/T_f}\), with \(T_f\) being the temperature at which the influence of FCQPT vanishes and \(a\) is a constant. Thus, the ordered state existing at \(T = 0\) is separated from the disordered state by first order phase transition. Hence, this ordered state cannot exist at any finite temperatures and is driven by the parameter \(\epsilon\) at \(x > x_{FC}\) the system is on the disordered side of FCQPT; at \(x = x_{FC}\), Eq. (1) possesses the non-trivial solutions \(n_0(p)\) with \(p_i = p_F = p_F\); at \(x < x_{FC}\), the system is on the ordered side.

Due to the first order phase transition, both at the FCQPT point and behind it there are no critical fluctuations accompanying second order phase transitions and suppressing the quasiparticles. As a result, the quasiparticles survive and define the thermodynamic and transport properties of HF systems. This is in agreement with recent facts obtained in measurements on CeCoIn\(_5\) \([7]\).

At \(T > 0\), the quasiparticle distribution is given by the Fermi function

\[
n(p, T) = \left\{1 + \exp \left[\frac{(\epsilon(p, T) - \mu)\pi}{\epsilon(p, T) - \mu}\right]\right\}^{-1}, \tag{3}
\]

where \(\epsilon(p, T)\) is the single-particle spectrum. It follows from Eq. (3) that \(n(p, T \to 0)\) tends to the step function \(\theta(p - p_F)\) if \(M^*\) is finite at \(T \to 0\) and we are dealing with LFL. Eq. (3) can be recast as

\[
\epsilon(p, T) - \mu(T) = T\ln \frac{1 - n(p, T)}{n(p, T)} \tag{4}
\]

As \(T \to 0\), the logarithm on the right hand side of Eq. (4) is finite when \(p \in (p_F - p_i)\) so that \(T\ln(-\cdot) \to 0\), and we again arrive at Eq. (1). Near the Fermi level the single-particle spectrum can be approximated as

\[
\epsilon(p \simeq p_F, T) - \mu \simeq \frac{p_F(p - p_i)}{M^*(T)}. \tag{5}
\]

At low temperatures, as it is seen from Eq. (5), the effective mass diverges as

\[
M^*(T) \simeq p_F\frac{p_F - p_i}{4T}. \tag{6}
\]

At \(T \ll T_f\), Eq. (5) is valid and describes the quasiparticles with the energy \(\epsilon(p)\) and the distribution function \(n_0(p)\). The energy \(\epsilon\) belongs to the interval

\[
\mu - 2T \leq \epsilon \leq \mu + 2T. \tag{7}
\]
3. Asymmetric conductance in HF metals and HTSC

When both metals are in their normal state, the tunneling current \( I(V) \) through the point contact between these two ordinary metals is proportional to the driving voltage \( V \), to the squared modulus of the transition amplitude \( t \) and does not depend on the density of states \([1,2]\)

\[
I(V) = 2|t|^2 \int f_n(\epsilon - V) - n_F(\epsilon) \, d\epsilon.
\]

(8)

Here, \( n_F(\epsilon) \) is the quasiparticle distribution function of ordinary metal. We use an atomic system of units: \( e = M = \hbar = 1 \), where \( e \) is electron charge. Since temperature is low, we approximate \( n_F(\epsilon) \) by the step function \( \theta(\epsilon - \mu) \). It is seen from Eq. (8) that \( I(V) = a_1 V \) and \( \sigma_d(V) = dI/dV = a_1 \), \( a_1 = \text{const} \). Thus, within the LFL theory the differential tunneling conductivity \( \sigma_d(V) \) is a symmetric function of the voltage \( V \).

In the case of the heavy electron liquid with FC, the tunneling current is of the form \([8]\)

\[
I(V) = \int [n(\epsilon - V, T) - n_F(\epsilon, T)] \, d\epsilon.
\]

(9)

Here, we have replaced the distribution function \( n_F(\epsilon) \) of ordinary metal by \( n(\epsilon, T) \) so that \( n(\epsilon, T) \to 0 \) \( \to n_0(\epsilon) \) where \( n_0(\epsilon) \) is the solution of Eq. (1) and also normalized the transition amplitude \( |t|^2 = 1 \). The differential conductivity, \( \sigma_d(V) = dI/dV \), is given by

\[
\sigma_d(V) = \frac{1}{T} \int n(\epsilon(z) - V, T)(1 - n(\epsilon(z) - V, T)) \frac{\partial z}{\partial z} \, dz,
\]

(10)

where \( z = p/p_F \). We take dimensionless momentum \( z \), instead of energy \( \epsilon \), as an independent variable, since the distribution function \( n_0 \) is a continuous function of \( z \) rather than of \( \epsilon \) as can be seen from Fig. 1. Indeed, the energy \( \epsilon \) is a constant in the range \( p_1 - p \) while the distribution function varies in this range. It follows from Eq. (10) that the asymmetric part \( \Delta \sigma_d(V) = (\sigma_d(V) - \sigma_d(0))/2 \) of the differential conductivity is of the form

\[
\Delta \sigma_d(V) = \frac{1}{2} \int \frac{\alpha(1 - \alpha^2)(1 - 2n(z, T))}{[n(z, T) + 1 - n(z, T)]\alpha[n(z, T)\alpha + 1 - n(z, T)]} \times \frac{\partial n(z, T)}{\partial z} \, dz,
\]

(11)

It is worth noting that according to Eq. (11) we have \( \Delta \sigma_d(V) = 0 \) if the considered HF metal is replaced by an ordinary metal. Indeed, the effective mass is finite at \( T \to 0 \) so that the integrand becomes an odd function of \( x = z - 1 \), while the limits of integration can be taken \(-\infty, \infty \) since the integrand behaves like \( \exp |x| \) at large \( |x| \). On the other hand, the integrand is no longer an odd function if the particle–hole symmetry is violated. As it is seen from Fig. 1, there are no reasons to expect that a Fermi liquid with FC conserves this symmetry. Thus, we conclude that the differential conductivity becomes an asymmetric function of the applied voltage for heavy electron liquid with FC.

To estimate \( \Delta \sigma_d(V) \), we observe that it is zero when \( V = 0 \) as it should be and follows from Eq. (11) as well. It is seen from Eq. (11) that at low voltage \( V \) the asymmetric part behaves as \( \Delta \sigma_d(V) \propto V \). Then, the natural scale to measure the voltage is \( 2T \), as it is seen from Eq. (7). In fact, the asymmetric part is proportional to \( (p_1 - p_1)/p_F \). As a result, we obtain

\[
\Delta \sigma_d(V) \simeq c \left( \frac{V}{2T} \right) \frac{p_l - p_1}{p_F} \simeq c \left( \frac{V}{2T} \right) S_0 \frac{1}{x}.
\]

(12)

Here, \( c \) is a constant of the order of unity. We conclude that asymmetric conductivity measurements can provide valuable information about the entropy \( S_0 \) which determines the divergence of the Grüneisen ratio \( \Gamma(T) \) as well since \( \Gamma(T) \propto S_0/\sqrt{T} \) \([9]\). So, we may conclude that STM and PCS techniques can give experimental evidence about the NFL behavior and thermodynamic properties of strongly correlated fermion systems. The constant \( c \) can be evaluated using the analytically solvable models. For example, calculations of \( c \) within a simple model, when Landau functional \( E[n(p)] \) is of the form \([10]\)

\[
E[n(p)] = \int \frac{p^2}{2M(2\pi)^3} + \frac{1}{2} \int V(p_1 - p_2)n(p_1)n(p_2) \frac{dp_1 dp_2}{(2\pi)^6},
\]

(13)

with the inter-particle interaction

\[
V(p) = g_0 \frac{\exp(-\beta_0 |p|)}{|p|},
\]

(14)

gives \( c \simeq 1/2 \). It follows from Eq. (12), that when \( V \simeq 2T \) and FC occupies a noticeable part of Fermi volume \( \sigma_0 \), then the asymmetric part becomes comparable with differential conductivity, \( \Delta \sigma_d(V) \sim \sigma_d(V) \).

The asymmetric behavior of the conductivity can be observed in measurements on both high-\( T_c \) metals in their normal state and the heavy fermion metals, for example, such as CeCoIn\(_5\) and YbRh\(_2(\text{Si0.95Ge0.05})_2\) which are expected to have undergone FCQPT. Indeed, the measurements on these metals have shown that the Grüneisen ratio \( \Gamma(T) \) diverges \([11,12]\). In that case, the electronic systems of these metals are to undergo FCQPT in order to make the temperature independent part \( S_0 \) of the entropy finite \([9]\). As a result, the asymmetric part of the conductance

\[
\Delta \sigma_d(V) \text{ becomes finite. We note that at sufficiently low temperatures, the application of magnetic field can restore the LFL behavior making the asymmetry of the tunneling conductivity vanish \([8]\). Therefore, the measurements have to be carried out when the corresponding HF metal demonstrates the NFL behavior.}

Point contact spectroscopy has recently been used to investigate the HF metal CeCoIn\(_5\). The dynamic conductance spectra shown in Fig. 2 have been obtained \([13]\). As a result, the asymmetric character of conductance has been observed in CeCoIn\(_5\) both in its superconducting and NFL normal states. Fig. 2 shows the conductance \( dI/dV \) as a function of voltage \( V \). The finite and even enhanced subgap conductance seen in Fig. 2 as well as that below the critical temperature \( T_c \) of superconducting...
phase transition arises from Andreev reflection, see e.g. [2]. It is seen from Fig. 2 that the asymmetric conductance develops at about 45 K, that is well above the critical temperature $T_c = 2.3$ K of superconducting phase transition. The asymmetry becomes more pronounced with decreasing temperature down to $T_c$. Then it remains almost the same down to 0.4 K [13].

In Fig. 3, we plot the results of our calculations (based on the functional (13) with the interaction (14)) of the asymmetric part of the conductance $\Delta \sigma_d(V)$ as a function of voltage when the heavy electron liquid is in its normal state. We normalize both the voltage $V$ and temperature $T$ by the Fermi energy $E_F$. We use the dimensionless coupling constant $g = (g_0 M)/(2\pi F)$ and $\beta = \beta_0 F$. FCQPT takes place when the parameters reach their critical values, $\beta = b_c$ and $g = g_c$. In the considered case, we take $\beta = 3$ (the corresponding $g = g_c = 6.7167$) and $g = 8$ so that $(p_f - p_i)/p_F \sim 0.1$. It is seen from Fig. 2 that asymmetric conductance vanishes at elevated temperatures when $T \rightarrow T_f$. The asymmetric part of the conductance $\Delta \sigma_d(V)$, extracted from the data of Fig. 2, is reported in the inset to Fig. 3. It is seen that our calculations are in good qualitative agreement with experimental data. Although we were not aiming to attain detailed quantitative agreement with the data within our simple model, a better agreement can be achieved by taking the appropriate Fermi energy value. The corresponding results will be published elsewhere.

The asymmetric conductivity $\Delta \sigma_d(V)$ can also be observed when both HTSC and HF metals in question go from normal to superconducting phase. The reason now is that $n_0(p)$ is again responsible for the asymmetric part of the differential conductivity measured by both STM and PSC. As it was shown in Ref. [4], the function $n_0(p)$ is not appreciably disturbed by the superconductive pairing interaction which is relatively weak as compared to Landau interaction forming the distribution function $n_0(p)$. Therefore, the asymmetric conductance remains approximately the same below $T_c$. This result is in good agreement with experimental facts as it is seen from the lower inset of Fig. 3. We also conclude that Andreev reflection can be considered as a useful effect when studying the asymmetric conductance and the NFL behavior. In that case, when calculating the tunneling conductance measured by scanning tunneling microscopy, we have to take into account that $n_s(E) = N(\varepsilon - \mu) E / \sqrt{E^2 - \Delta^2}$ (15) comes now into play since the density of states $N_s(E)$ of the superconducting metal is zero in the gap, i.e. when $|E| \leq |\Delta|$. Here, $\Delta$ is the superconducting gap at $T = 0$, $E$ is the quasiparticle energy in the superconducting state, related to the normal state quasiparticle energy as $\varepsilon - \mu = \sqrt{E^2 - \Delta^2}$. It follows from Eq. (15) that the tunneling conductance can be asymmetric if the density of states $N(\varepsilon - \mu)$ is asymmetric with respect to the Fermi level [14] as it is in the case of Fermi systems with FC. Now we can adjust Eq. (12) for the case of superconducting HF metal, multiplying the right hand side of this expression by $N_s/N(0)$ and replacing the quasiparticle energy $\varepsilon - \mu$ by $\sqrt{E^2 - \Delta^2}$ with $E$ being substituted by the voltage $V$. As a result, Eq. (12) can be presented in the following form:

$$\Delta \sigma_d(V) \simeq \frac{V}{\Delta} \left| \frac{\sqrt{V^2 - \Delta^2}}{\sqrt{V^2 - \Delta^2}} \frac{p_f - p_i}{p_f} \right| \sim \frac{V S_0}{\Delta^2}.$$ (16)

We note that the entropy $S_0$ on the right hand side of Eq. (16) is a temperature independent part of the normal (rather than superconducting) state entropy. Thus, as it follows from Eq. (16), the entropy $S_0$ characterizing the normal state can be evaluated measuring the asymmetric part of tunneling conductance of the superconducting state. We note that the scale $2T$ entering Eq. (12) is replaced by the scale $\Delta$ in Eq. (16). In the same way, as Eq. (12) is valid up to $V \sim 2T$, Eq. (16) is valid up to $V \sim 2|\Delta|$ and up to temperatures $T \leq T_c$. It is seen from Eq.
(16) that the asymmetric part of the differential tunneling conductance becomes as large as the entire differential tunneling conductance at \( V \sim 2|\Delta| \) under the condition that FC occupies a large part of the Fermi volume \((p_f - p_i)/p_F \simeq 1\). In the case of a \( d \)-wave (or other unconventional) pairing, the right hand side of Eq. (16) has to be additionally integrated over the spatially inhomogeneous part of a corresponding gap function. As a result, \( \Delta \sigma_{d}(V) \) is expected to be finite even at \( V = \Delta_1 \), where \( \Delta_1 \) is the maximal value of the \( d \)-wave gap function, while in the case of \( s \)-wave pairing Eq. (16) is valid at \( V \geq \Delta \). In the case of PCS due to Andreev reflection the asymmetrical conductance can be observed even at \( V \leq \Delta \). STM measurements performed on underdoped Bi\(_{2}\)Sr\(_2\)CaCu\(_2\)O\(_8\) single crystal have shown that the asymmetric conductivity persists up to room temperature that is well above \( T_c = 83.0 \) K, see Fig. 19 of Ref. [2]. This behavior is compatible with the behavior of the asymmetric conductance in Fig. 2. Therefore, we can conclude that the asymmetric part of the conductance described by Eqs. (12) and (16) is in good qualitative agreement with the experimental facts published in Refs. [2, 13], while the asymmetry starts from FCQPT yielding the FC state with the particle–hole symmetry violation and making the density of states be asymmetric with respect to the Fermi level.

4. Conclusions

We have shown that scanning tunneling microscopy and point contact spectroscopy being sensitive to both the density of states and the quasiparticle occupation numbers are ideal techniques for studying the effects related to a violation of the particle–hole symmetry. We have demonstrated that the asymmetric part of the conductance can be observed when HF metals and high-\( T_c \) superconductors are normal and/or superconducting. Our theoretical results are in good agreement with available experimental data. Since in the pure LFL case the particle–hole symmetry is conserved and both the differential tunneling conductivity and dynamic conductance are symmetric functions of voltage bias, the measurements of the asymmetric part of the conductance can be viewed as a powerful tool to investigate the NFL behavior of strongly correlated Fermi systems. We have derived that the asymmetric part is defined by a temperature independent part of the entropy characterizing the normal state of the electronic system of a strongly correlated metal. As a result, it became possible to study the thermodynamic properties measuring the transport characteristics of a substance. Thus, STM and PCS provide a new and very useful direction in the experimental studies of the NFL behavior of HTSC and HF metals. Our consideration of the conductance asymmetry suggests that FCQPT is intrinsic to strongly correlated substances, which can be viewed as the universal cause of the non-Fermi liquid behavior observed in different metals.

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